1 Module 1: Sample Space and Probability

1. If $P(A \cup B) = 0.7$ and $P(A \cup B^c) = 0.9$, then $P(A)$ is given by

   A. 0.2

   B. 0.4

   C. 0.6

   D. 0.8

   Answer: C

2. A sample space consists of five simple events E_1, E_2, E_3, E_4 and E_5. If $P(E_1) = P(E_2) = 0.15$, $P(E_3) = 0.4$ and$P(E_4) = 2P(E_5)$, then the probability of E_4 and E_5 is:

   A. $(0.3, 0.2)$

   B. $(0.2, 0.3)$

   C. $(0.1, 0.3)$

   D. $(0.2, 0.1)$

   Answer: D

3. If $P(D|F) > P(E|F)$ and $P(D|F^c) > P(E|F^c)$, then the relation between $P(D)$ and $P(E)$ is

   A. $P(D) > P(E)$

   B. $P(D) < P(E)$

   C. $P(D) = P(E)$

   D. cannot be determined

   Answer: A

4. If $P(A|B) = P(A|B^c)$, then the events A and B are

   A. Dependent events

   B. Mutually exclusive events

   C. Independent events

   D. Exhaustive events

   Answer: C

5. Suppose the sample space is $\Omega = \{1, 2, 3, 4\}$ and all sample points are equally likely. Suppose $A = \{1, 2\}$, $B = \{1, 3\}$ and $C = \{1, 4\}$. Then the events A, B and C are

   A. mutually independent

   B. neither pairwise independent nor mutually independent

C. pairwise independent but not mutually independent

D. not pairwise independent

Answer: C

2 Module 2: Distribution function of a random variable

1. The probability distribution function of a random variable X is given by

$$F_X(x) = \begin{cases} 0, & x < -1; \\ 1/4, & -1 \leq x < 0. \\ 1/2, & 0 \leq x < 1; \\ x/2, & 1 \leq x < 2. \\ 1, & x \geq 2; \end{cases}$$

Then

A. The median is 0

B. Every point in $[0, 1]$ is a median

C. The median is 1

D. Every point in $(1, 2]$ is a median

Answer: B

2. Let X be random variable with distribution function $F(x) = \begin{cases} 0, & x < 0; \\ \frac{1}{2}, & x = 0. \\ \frac{1}{2} + \frac{x}{4}, & 0 < x < 1; \\ 1, & x \geq 1; \end{cases}$

Then

A. $P[0 \leq X < \frac{1}{2}] = \frac{5}{8}$ and $P[X > 1] = 0$

B. $P[0 \leq X < \frac{1}{2}] = \frac{1}{8}$ and $P[X \leq 1] = \frac{3}{4}$

C. $P[0 \leq X \leq \frac{1}{2}] = \frac{5}{8}$ and $P[X \leq 1] = \frac{3}{4}$

D. $P[0 \leq X < \frac{1}{2]} = \frac{5}{8}$ and $P[X < 1] = \frac{3}{4}$

Answer: A

3. The distribution function of a random variable X is given below

$$F(x) = \begin{cases} 0, & x < 0; \\ \frac{1}{4}, & 0 \leq x < 1; \\ \frac{1}{2} + \frac{1}{2}(1 - exp-(x-1)), & x \geq 1; \end{cases}$$

Then the probabilities $P[0 \leq X < 1]$ and $P[\frac{1}{2} < X \leq 1]$ respectively are

A. 0 and $\frac{1}{4}$

B. $\frac{1}{4}$ and $\frac{1}{2}$

C. $\frac{1}{4}$ and $\frac{1}{4}$

D. 0 and $\frac{1}{2}$

Answer: C

4. Let the random variable X has the distribution function

$$F(x) = \begin{cases} 0, & x < 0; \\ \frac{x}{2}, & 0 \le x < 1; \\ \frac{3}{5}, & 1 \le x < 2; \\ \frac{1}{2} + \frac{x}{8}, & 2 \le x < 3; \\ 1, & x \ge 3; \end{cases}$$

Then $P[2 \le X < 4]$ is equal to

A. 2/5

B. 3/5

C. 1

D. 1/2

Answer: A

5. Consider a measurable space $(\Omega, \mathcal{F})$, where $\Omega = \{1, 2, 3, 4\}$ $A = \{1, 2\}$ and $\mathcal{F} = \{\emptyset, A, A^c, \Omega\}$ and define functions:

$X(\omega) = 1 \, \forall \omega \in \Omega$

$$Y(\omega) = \begin{cases} 0, & \omega \in A; \\ 1, & \omega \notin A. \end{cases}$$

$Z(\omega) = \omega \, \forall \omega \in \Omega$

Then which of the following is true?

A. All the functions X, Y, Z are random variables

B. Only X and Y are random variables

C. Only X is a random variable

D. Only Y is a random variable

Answer: B

3 Module 3: Types of Random Variables and Expectation

1. Let $F(X)$ be a cdf of a random variable X, where

$$F(x) = \begin{cases} 0, & x < 0; \\ \frac{x+1}{3}, & 0 \le x < 1; \\ 1, & x \ge 1; \end{cases}$$

Then the variance of X is given by

A. 1/9

B. $1/36$

C. $1/6$

D. $7/36$

Answer: D

2. Let $P[X_1 = 2, X_2 = 3] = 1$, then $E(X_1), E(X_2), V(X_1)$ and $V(X_2)$ are as follows

A. (1,2,3,1)

B. (2,3,1,1)

C. (2,3,0,0)

D. (1,1,2,3)

Answer: C

3. Suppose $P(X = x+1) = \frac{p(n-x)}{q(x+1)} P(X = x)$. Then the probability mass function of X is given by

A. $P(X = x) = p^n(1 - q^n)^x \quad x = 0, 1, ...$

B. $P(X = x) = e^{-np}(np)^x/x! \quad x = 0, 1, ...$

C. $P(X = x) = \binom{n}{x} p^x (q)^{n-x} \quad x = 0, 1, ...$

D. $P(X = x) = pq^{x-1} \quad x = 0, 1, ...$

Answer: C

4. Let the random variable X has the distribution function

$$F(x) = \begin{cases} 0, & x < 0; \\ \frac{1}{4}, & 0 \leq x < 1; \\ \frac{1}{3}, & 1 \leq x < 2; \\ \frac{1}{2}, & 2 \leq x < \frac{11}{3}; \\ 1, & x \geq \frac{11}{3}; \end{cases}$$

Then $E(X)$ is equal to

A. $2/5$

B. 2.25

C. 1

D. 3

Answer: B

5. Let X and Y be two random variables having the joint probability density function

$$f(x, y) = \begin{cases} 2, & 0 < x < y < 1; \\ 0, & otherwise; \end{cases}$$

Then the conditional probability $P\left(X \leq \frac{2}{3} | Y = \frac{3}{4}\right)$ is equal to

A. $5/9$

B. 2/3

C. 7/9

D. 8/9

Answer: D

4 Module 4: Discrete Probability Distributions

1. Suppose X_1 and X_2 are independent Bernoulli random variables with $E(X_1) = p_1$ and $E(X_2) = p_2$. Then X_1X_2 follows

   A. Bernoulli distribution with mean p_1p_2

   B. Bernoulli distribution with mean $p_1 + p_2$

   C. Binomial distribution with $n = 2$ and mean $p = p_1p_2$

   D. Bernoulli distribution with mean $(1 - p_1)(1 - p_2)$

   Answer: A

2. A basketball player makes 70% of her free throws. She takes 7 free throws in a game. If the shots are independent of each other, the probability that she makes 5 out of the 7 shots is about

   A. 0.635

   B. 0.318

   C. 0.015

   D. 0.329

   Answer: B

3. In 1989 Newsweek reported that 60% of young children have blood lead levels that could impair their neurological development. Assuming that a class in a school is a random sample from the population of all children at risk, the probability that more than 3 children have to be tested until one is found to have a blood level that may impair development is

   A. 0.064

   B. 0.096

   C. 0.64

   D. 0.16

   Answer: A

4. In a Poisson Distribution, if n is the number of trials and p is the probability of success, then the mean value is given by

A. np

B. $(np)^2$

C. $np(p-1)$

D. p

Answer: A

5. Distribution whose function is calculated by considering Bernoulli trials that are infinite in number is classified as

A. negative Poisson distribution

B. bimodal cumulative distribution

C. common probability distribution

D. negative binomial probability distribution

Answer: A

5 Module 5: Continuous Probability Distributions

1. The heights (in inches) of males in the United States are believed to be Normally distributed with mean p. The average height of a random sample of 25 American adult males is found to be $\bar{x} = 69.72$ inches, and the standard deviation of the 25 heights is found to be $s = 4.15$. The standard error of $\bar{x}$ is

A. 0.17

B. 0.69

C. 0.83

D. 1.856

Answer: C

2. Which of the following is not a characteristic of the normal distribution?

A. it is a symmetrical distribution

B. the mean is always zero

C. the mean, median and mode are equal

D. it is a bell-shaped distribution

Answer: B

3. Suppose a flight is about to land and the announcement says that the expected time to land is 30 minutes. Find the probability of getting flight land between 25 to 30 minutes?

A. 1/2

B. 0

C. 5/6

D. 1/6

Answer: D

4. The number of miles that a particular car can run before its battery wears out is exponentially distributed with an average of 10,000 miles. The owner of the car needs to take a 5000-mile trip. What is the probability that he will be able to complete the trip without having to replace the car battery?

A. 0.604

B. 0.1

C. 0.9

D. 0.3

Answer: A

5. Which of the following distribution has memory less property

A. Normal distribution

B. Gamma distribution

C. Exponential distribution

D. Beta distribution

Answer: C