Swami Ramanand Teerth Marathwada University,

Vishnupuri, Nanded. (Maharashtra), India.

M.A. /M. Sc. (Mathematics) Syllabus Semester-IV

(For Campus School)

SCHOOL OF MATHEMATICAL SCIENCES

M.Sc. (Mathematics)-II year (CGPA)

Paper No.	Name of the paper
MTU-401	Boundary Value Problems
MTU-402	Integral Equations and transforms
Any three papers from MTU-403 to MTU-416 which will be taught in the School	
MTU-403	Probability Theory
MTU-404	Algorithms and their analysis.
MTU-405	Commutative Algebra
MTU-406	Classical Mechanics
MTU-407	Theory of Relativity
MTU-408	Representation Theory of finite groups
MTU-409	Algebraic Topology
MTU-410	Difference Equations
MTU-411	Fuzzy sets and their applications
MTU-412	Advanced Functional Analysis.
MTU-413	Algebraic Number Theory.
MTU-414	Computational Geometry
MTU-415	Fluid Dynamics
MTU-416	Baer * rings.
MTU-417 (Compulsory)	LATEX and Project work.

(For entire course structure and evaluation strategy, one may visit page number 6, from syllabus of M.Sc. (Mathematics)-I sem)

MTU-401: BOUNDARY VALUE PROBLEMS

(Maximum Number of Periods: 60)

Unit 1: Fourier Series:

Piecewise Continuous Functions, Fourier Cosine Series, Fourier Sine Series, Fourier series, adaptations to other intervals.

Unit 2: Convergence of Fourier Series:

One-Sided Derivatives, A Property of Fourier Coefficients, Two lemmas, A Fourier Theorem, Discussion of the theorem and its corollary, convergence on other intervals, A Lemma, Absolute and Uniform Convergence of Fourier Series, Differentiation of Fourier Series, Integration of Fourier Series.

Unit 3: Partial Differential Equations of Physics:

Linear Boundary Value Problems, One-dimensional Heat Equation, Laplacian in cylindrical and spherical coordinates, derivations, Boundary conditions, A vibrating string, vibrations of bars and membranes, general solution of the wave equation, types of equations and boundary conditions.

Unit 4: The Fourier Method:

Linear Operators, Principle of Superposition, a temperature problem, a vibrating string problem.

Unit 5: Boundary Value Problems:

A slab with faces at prescribed temperatures, a slab with internally generated heat, steady temperatures in a rectangular plate, cylindrical coordinates, a string with prescribed initial conditions, resonance, an elastic bar.

Unit 6: Fourier Integrals and applications:

The Fourier Integral Formula, Dirichlet's integral, Two lemmas, A Fourier integral theorem, The Cosine and Sine integrals, more on superposition of solutions.

Unit 7: Orthonormal Sets:

Inner products and orthonormal sets, Generalized Fourier series, Best approximation in the mean, Bessel's inequality and Parseval's equation, applications to Fourier series.

Unit 6: Sturm-Liouville Problems and applications:

Regular Sturm-Liouville Problems, modifications, orthogonality of eigen functions, real-valued eigen functions and non-negative eigen values, methods of solution.

Text Book: - R.V. Churchill and J. Brown.: "Fourier Series and Boundary Value Problems"

(7_{th} edition)(Publisher: McGraw-Hill Book Company)

Scope: - Chapter 1 to 8, Chapter 9- articles 1 to 5.

Reference Books:

i) W. E. Boyce and R. C. DiPrima, "Elementary Differential Equations and Boundary Value Problems", John Wiley and Sons.(7^{th} Edition)

D.G.Zell and Cullen, "Differential Equations with Boundary Value Problems", Cengage Learning Publishers.

MTU-402: INTEGRAL EQUATIONS AND TRANSFORMS

(Maximum Number of Periods: 60)

Unit 1 : - Introduction

Types of integral equation, Types of Kernels ,Eigen values and eigen functions, Abel problem, differentiation under the integral sign, relation between differential equation and integral equation, boundary value problems.

Unit 2:- Fredholm Integral equations

Solution of Homogeneous Fradholm integral equation of the second kind with separable kernel, orthogonality of eigen functions, Non-homogeneous Ferdholm integral equation with separable kernel.

Unit 3 :- Hilbert-Schmidt theory

Introduction, Orthogonal system of functions, Gram-Schmidt orthogonalization process, Riesz-Fischer theorem, symmetric kernel, Hilbert-Schmidt theorem, solution of the Fredholm integral equation of first kind, Schmidt solution of the Non-homogeneous Fredholm integral equation of second kind.

Unit 4 :- Solution of integral equation

Solution of the Volterra integral equation of second kind by successive substitution method, Solution of the Fredholm integral equation by successive substitution method, Solution of the Fredholm integral equation by successive approximation ,reciprocal functions, Volterra's solution of Fredholm equation.

Unit 5 :- Singular integral equation

Solution of Abel's integral equation, weakly singular kernel, Cauchy principal for integrals, Cauchy type integrals, Hilbert kernel, and Hilbert formula.

Unit 6 :- Fourier Transform

Definition, Properties evaluation of Fourier and inverse Fourier transforms of functions, Convolution theorem for FourierTransform, Sine and Cosine Fourier transforms, Solving differential equations and integral equations using Fourier Transform

Unit 7:-Laplace Transform

Definition , Properties, evaluation of Laplace and Inverse Laplace transforms of functions, Convolution theorem for Laplace Transforms, Solving initial value problem using Laplace Transforms, Solving integral equation using Laplace Transforms.

Reference Books:

1] A.M. Wazwaz:"A First course in integral equations ", (world Scientific)
2]Ram P.Kanwal:"Linear Differential Equation"

3]A.J. Jerri:" Introduction to Integral Equation with Applications", (2^{nd} edition)

Wiley Interscience

MTU-403: PROBABILITY THEORY

(Maximum Number of Periods: 60)

UNIT 1:

Probability, Probability measure on a s-field, Probability space (definition only), Properties of probability measure. Caratheodory Extension theorem (Statement only), Lebesgue-Stieltjes measure, Fatou's lemma, Jensen's inequality, Fubini's theorem.

UNIT 2:

Characteristic Functions, Moment Generating Functions, properties, inequalities, uniqueness theorem, Inversion theorem, continuity theorem. Distribution functions and its properties, convergence of sequence of random variables, convergence almostsure, convergence in probability, convergence in distribution, convergence in r^{th} mean, inter relations between different types of convergences.

UNIT 3:

Independence and Convolution, Weak Law of Large Numbers ,Strong Limit Theorems , Series of Independent Random variables, Strong Law of Large Numbers ,Central Limit Theorem, Accompanying Laws, Infinitely Divisible Distributions, Laws of the iterated logarithm. Chebyshev's weak law of large numbers, Khinchin's weak law of large numbers, Kolmogorov's strong law of large numbers (statement only), Kolmogorov's inequality.

UNIT 4:

Conditioning, Conditional Expectation, Conditional Probability, Markov Chains. Stopping Times and Renewal Times, Countable State Space.

UNIT 5:

Definitions and properties, Martingale Convergence Theorems, Doob Decomposition Theorem, Stopping Times, Upcrossing Inequality, Martingale Transforms, Option Pricing, Martingales and Markov Chains.

Ergodic Theorems, Structure of Stationary Measures, Stationary Markov Processes, Mixing properties of Markov Processes, Central Limit Theorem for Martingales, Stationary Gaussian Processes. Optimal Control, Optimal Stopping, Filtering.

Text Book:

Vardhan S. R. S. "Probability Theory", New York University Press.

Scope: - Chapter 1 to Chapter 7.

Reference Books:

1) Ash Robert (1972) Real analysis and probability, Academic press.
2) Mukhopadhy P. (2002) Theory of Probability, New central book agency, Calcutta.
3) Bhat B. R. (2000) Modern Probability Theory, New age International.
4) Billingsley P. P. (1986) probability and measure, Wiley.
5) Dudewicz E. J. and Mishra S. N. (1988) Modern Mathematical Statistics, Wiley Int. student's Edition.
6) Rohatgi V. K. (1984) An Introduction to Probability Theory and Mathematical Statistics, Wiley Eastern.

MTU-404: ALGORITHMS & THEIR ANALYSIS

(Maximum Number of Periods: 60)

Algorithms; Assignment, arithmetic, relational, logical operators; Truth tables; input/output statements; conditional statements; iterative statements; functions; recursion, recursion trees, binary trees ; worst-case, best-case, average-time requirements; recurrence relations; P,NP,NP-completeness, NP-Hard problems; lower bound of time requirements.

Algorithms to be discussed:

1. Min, max, average, standard deviation.
2. Linear and binary search(iterative and recursive)
3. Simple sorts: selection and bubble
4. Factorial(iterative and recursive)
5. Fibonacci(iterative and recursive)
6. Tower of Hanoi(iterative and recursive)
7. Merging sorted lists.
8. NlogN sorts: Merge sort and heap sort.
9. Matrix multiplication: Iterative O (n^3) versus Strassen's recursive algorithm O (n^2.807). A brief look at Coppersmith–Winograd algorithm O(n^2.376).
10. Prime numbers.

Books:

(1) S. Lipschutz, ”Data Structures”, Schaum’s Outline series.

(2) Dino Mandrioli, Carlo Ghezzi “Theoretical Foundations of Computing Science”, Wiley, 1987

MTU-405: COMMUTATIVE ALGEBRA

(Maximum Number of Periods: 60)

Unit 1: Rings and Ideals (Pre-requisites):

Rings and ring homomorphisms, ideals, quotient rings, zero divisors, nilpotent elements, units, prime ideals and maximal ideals, nilradical and Jacobson radical, operations on ideals, extension and contraction.

Unit 2: Modules:

Modules and module homomorphisms, submodules and quotient modules, operations on submodules, Direct sum and product, finitely generated modules, exact sequences, Tensor product of modules, Restriction and extension of scalars, Exactness properties of the tensor product, Algebras, Tensor product of algebras.

Unit 3: Rings and Modules of Fractions:

Local properties, Extended and contracted ideals in ring of fractions, Primary Decomposition.

Unit 4: Integral Dependence and Valuations:

Integral dependence, the going-up theorem, integrally closed integral domains, the going-down theorem, chain conditions.

Unit 5: Noetherian and Artinian Rings:

Primary decomposition in Noetherian rings, Artin rings.

Unit 6: Discrete Valuation Rings and Dedekind Domains:

Discrete valuation rings, Dedekind domains, fractional ideals.

Text Book: - M. F. Atiyah and I. G. Macdonald, "Introduction to Commutative Algebra" Addison-Wesley Publishing Company.

Scope: - Chapter 1 to Chapter 9.

Reference Books:

i) H.Matsumura,"Commutative Ring Theory", Cambridge University Press.
ii) N. S. Gopalakrishnan, "Commutative Algebra".
iii) D.S. Dummite and R.M. Foote, "Abstract Algebra" Second, John Wiley & Sons.
iii) D.P. Patil, Patil, Storch," Introduction to Algebraic Geometry and Commutative Algebra", Anshan Publishers.
iv) S. Lang, "Algebra", Springer(GTM).

MTU: 406 CLASSICAL MECHANICS
(Maximum Number of Periods: 60)

Unit 1:

Mechanical of system of particles, Mechanics of system of particles, Conservation theorems conservative forces with examples, Constraints, Generalized co-ordinates. D. Alembert's principle, Lagrange's equations of motion. The forms of Lagrange's equations of motion for non conservative systems and partially conservative and partially non conservative systems.
Kinetic energy as a homogeneous function of generalized velocities.
Simple applications of the Lagranian formulation.

Unit 2: Cyclic co-ordinates and generalized momentum conservation Theorems, Calculus of variation, Euler Lagrange's equation, First integrals of Euler Lagrange's equation, the case of several dependent variables, Geodesics in a plane, the minimum surface of revolution, Brachistochrome problem.
Isoperimetric problems, problems of maximum enclosed area.

Unit 3: Hamilton's Principle for conservative systems, Lagrange's equation from Hamilton principle for conservative system, Hamiltonian function. Hamiltonian canonical equations of motion, Derivation of Hamiltonian equation from variational principle, Physical significance of Hamiltonian, The principle of least action, Jocobi's form of the least action principle, Cyclic co-ordinates and Routh's procedure.

Unit 4: The independent co-ordinates of a rigid body, Orthogonal transformations, Properties of transformation matrix, Infinitesimal rotations, The Eulerian angles, The Calyley-Klein parameters, Eulers theorem on motion of rigid body, Angular momentum and kinetic energy of motion of a rigid body about a point.

Books:

i] Goldstein H. “Classical Mechanics”, Narosa Publishing House.
(Second edition) (Third Edition)

Reference Books:

1) Goldstein Pooler & Saflco Classical Mechanics Pearson Educations.
2) N.C. Rana & P. S. Jog Classsical Mechanics,
Tata Mc.Graw Hill (1992)

MTU: 407 THEORY OF RELATIVITY
(Maximum Number of Periods: 60)

Unit I:
Review of the special theory of relativity and the Newtonian theory of gravitation, Galilean and Lorentz transformations. Distinction between Newtonian space and relativistic space. The conflict between Newtonian Theory of gravitation and Special Relativity. The action principle, the energy momentum tensor.

Unit-II :
The stress energy momentum tensor for Incoherent matter, perfect fluid, electromagnetic field. Einstein's relativity: SR to GR, Non-Euclidean space time. General Relativity and gravitation, desirable features of gravitational theory. Principle of equivalence, Principle of covariance and Mach's Principles,

Unit-III :
Einstein's field equations, Derivation of Einstein's field equations from action principle, Flat space and empty space. Local conservation laws associated with perfect fluid distribution. Newtonian approximation: Relation between g_{44} and V, Einstein equations compared with Poisson equation.

Unit-IV :
Spherical symmetry, Einstein field equations under spherical symmetry. Schwarzschild exterior solution and its isotropic form, planetary orbits, General relativistic Kepler problem, Three crucial tests for general Theory of relativity: 1. Perihelion of the planet Mercury, 2. Bending of light, 3. Gravitational red shift, Schwarzschild interior solutions

References :

(1) Introduction to General Relativity - Ronald Ader , Maurice Bazin, Menahem Schiffer , 2 Edtion, McGraw Hill Company .
(2) Lectures of Relativity - T .M.Karade, et al Einstein Foundtion International, Nagpur .
(3) General Relativity and Cosmology - J.V .Narlikar , Macmillan Company of India, 1978.
(4) Gravitation and Cosmology :Principles and Applications of General Theory of Relativity - Steven Weinberg, John Wiley Publication.
(5) Relativity , Thermodynamics and Cosmology - R.C. Tolman (Oxford Press)
(6) Mathematical Theory of Relativity - A.S. Eddington, Cambridge University Press, 1965.
(7) Dr. S. R. Roy, Dr. Raj Bali, Theory of relativity, Jaipur publishing house
(8) . H. Stepheni: General Relativity: An Introduction to the theory of gravitational field. Cambridge University Press. (1982)
(9) R. Resnicik, Introduction to special relativity, Wiley Eastern Ltd

MTU: 408 REPRESENTATION THEORY OF FINITE GROUPS
(Maximum Number of Periods: 60)

Unit I: Representations, Subrepresentations, Tensor products, Symmetric and Alternating Squares.

Unit II: Characters, Schur's lemma, Orthogonality relations, Decomposition of regular representation, Number of irreducible representations, canonical decomposition and explicit decompositions. Subgroups, Product groups, Abelian groups. Induced representations.

Unit III: Examples, Cyclic groups, alternating and symmetric groups.

Unit IV: Integrality properties of characters, Burnside's $p^a q^b$ theorem, The character of induced representation, Frobenius Reciprocity Theorem, Meckey's irreducibility criterion,Examples of induced representations, Representations of supersolvable groups.

Textbook / Reference book:

1) W. Fulton, J. Harris, Representation Theory: A first course, Graduate Texts in Mathematics. Readings in Mathematics 129. Springer Verlag (1991). International Edition (Low priced Ed.)
2) M. Burrow, Representation Theory of Finite Groups, Academic Press, 1965.
3) N. Jacobson, Basic Algebra II, Hindustan Publishing Corporation, 1983.
4) S. Lang, Algebra, 3rded. Springer (India) 2004.
5) J.P. Serre, Linear Representation of Groups, Springer-Verlag, 1977
6) C. Musili, Representations Finite Groups, Hindustan Book Agency, 1993.
7) E. B. Vinberg: Linear Representations of Groups, Birkhäuser (1988).